{ B.Sc Part I (Physics Hons)
Dr Satyadeo Narayan Singh
S.B. College, Ara }

①

Question:- (a) On the basis of Lorentz transformation derive an expression for length contraction.

OR

Prove that the length of an object in a frame moving w.r. to an observer is smaller than the length of the same object as observed in stationary form.

(a) Define proper length

(b) A circle and a square are moving along x-axis. How will they appear to a stationary observer.

OR

Derive Lorentz-Fitzgerald contraction.

Ans:→

(a) Lorentz transformations:→

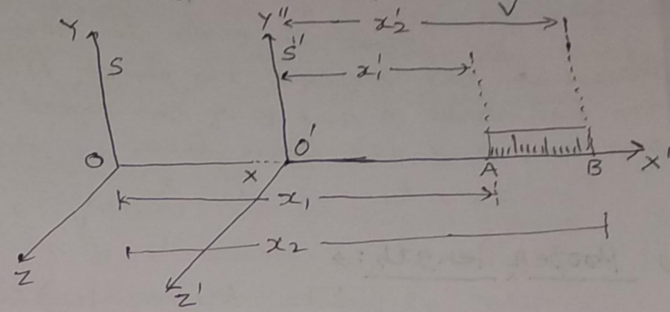
If two intertial system S and S'' are in relative motion so that S and S'' moves with a uniform velocity V to the right along the X-axis relative to S and (x, y, z, t) and (x', y', z', t') are the space and time co-ordinates of an event P in the inertial systems S and S'' respectively, then according to Lorentz transformation equations

$$x' = \frac{x - vt}{\sqrt{1 - V^2/c^2}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{V}{c^2}x}{\sqrt{1 - \frac{V^2}{c^2}}}$$



Length contraction:→ In classical physics the length of an object is the same for all observers irrespective of their velocities relateive to object.

According to this theory of relativity, the length of an object depends upon the velocity of the observer w.r. to object.

Suppose the observers O and O' in the inertial systems S and S'' are at rest w.r. to each other with their ~~original~~ origins coinciding at some time. They compare their measureing rods and find that both give the same values for the length of an object. AB and let it be = Lo. Now S'' moves to the right along the X-axis with the uniform velocity V w.r. to S with the object AB lying along its x - axis. The observer in the system S'' places his measuring rod along the x-axis with its left end at $A = x_1'$ and the right end $B = x_2'$

$\therefore$ For S'' $AB = x_2' - x_1' = L_0$

Let us now find the length of the object AB moving with the frame S'' as measured by the observer O in the frame S. Suppose the position of the object AB in the reference frame S is x_1 for the end A and x_2 for the end B, then the length of the object lying in the moving frame as measured by S

$$= x_2 - x_1 = L \text{ (~~sed~~ say)}$$

As S' is moving to the right along the X-axis with a velocity V with respect to S, therefore according to Lorentz transformations

$$x_1' = \frac{x_1 - Vt_1}{\sqrt{1-\frac{V^2}{c^2}}} \quad \text{and} \quad x_2' = \frac{x_2 - Vt_2}{\sqrt{1-\frac{V^2}{c^2}}}$$

$$\therefore x_2' - x_1' = \frac{(x_2 - x_1) - V(t_2 - t_1)}{\sqrt{1-\frac{V^2}{c^2}}} \quad \text{......} \quad (1)$$

Now in order to say that S may be able to measure the length of the object AB of S'', he should mark the two ends A and B of the object simultaneously, i.e at one end and the same time.

$$t_2 = t_1 \qquad \therefore x_2' - x_1' = \frac{x_2 - x_1}{\sqrt{1-\frac{V^2}{c^2}}}$$

(b) <u>Proper length</u>:→

The proper length of an object is defined as the length measured by a scale at rest w.r. to object.

In relation (1)

$x_2' - x_1'$ is actual or proper length of object as measured by S'' himself and is equal to L_0. The observed length of the same object as measured by S when S'' is in motion is equal to $x_2 - x_1 = L$

$$\therefore L_0 = \frac{L}{\sqrt{1-\frac{V^2}{c^2}}} \quad \text{or} \quad L = L_0\sqrt{1-\frac{V^2}{c^2}}$$

Hence when measuring the length of an

object in the system S'' moving with velocity v w.r to an observer in the system S, the observer finds the length to be contracted by the factor $\sqrt{1-\frac{V^2}{c^2}}$. The contraction is reciprocal, i.e the observer in the system S'' also finds the length of an object lying in the system S whose proper length is Lo to be equal to

$$L = L_0\sqrt{1-\frac{V^2}{c^2}}$$

This phenomenon is known as length contraction or space contraction. As it was suggested by ~~lort~~ Lorentz and Fizgerald. It is also called Lorentz-Fitzgerald contraction.

Thus according to the theory of relativity, the length of an object is not absolute but depends upon the relative motion of the object w.r.to the observer. It is maximum when the object is at rest in the observer's intertial system. In order to the length of an object measured in a frame moving with respect to an observer is smaller than the length of the same object observed in stationary frame.

(c) **Square and circle in motion :→**

When an intertial frame S' is moving with a velocity $\vec{V}$ along the x-axis w.r to an intertial fram S'', then according to relation ①

$$x_2 - x_1 = (x_2' - x_1')\sqrt{1-\frac{V^2}{c^2}}$$

or $$L = L_0\sqrt{1-\frac{V^2}{c^2}}$$